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Introduction

"Relational concepts" means different things to various investigators. Fortunately, any experimental task and procedure used by any animal research investigator can be reduced to the fundamental ways that Bourne (1970) and Thomas (1980) defined relational concepts. Bourne based all concepts on five basic logical operations and their complements, and he used tasks that were constructed consistently with the truth table that defined each of the five primary and complementary logical operations. Bourne referred to the five primary and complementary truth-table-based operations as "rules," and that term will be used here as Bourne used it. In order from most basic to most complex and with the primary listed first and the complementary listed second here, the five basic pairs of rules are (a) Affirmation and Negation, (b) Conjunctive and Alternative Denial, (c) Inclusive Disjunction and Joint Denial), (d) Conditional and Exclusion, (e) Biconditional and Exclusive Disjunctive.

Thomas (1980) drew heavily from Bourne (1970), and Gagné (1970), to construct a hierarchy of eight fundamental types of learning. Any and all learning tasks and procedures, no matter how complex, can be reduced to one of or combinations of the eight types in the hierarchy Thomas (1980) constructed; see Thomas (1996) and Bailey, McDaniel and Thomas (2007) for slight modifications to the 1980 hierarchy. The eight types and their levels in Thomas's hierarchy (1980) are (a) Level 1: Habituation and its opposite, Sensitization, (b) Level 2: Classical or Pavlovian Conditioning, (c) Level 3: S-R Learning also known as Operant Conditioning, (d) Level 4: Chaining of S-R units,(e) Level 5: Concurrent Discrimination Learning, (f) Level 6: Class Concepts, (g) Level 7: Relational Concepts I and (h) Level 8: Relational Concepts II. Following Gagné (1970), that the types of learning are hierarchical is based on lower levels being prerequisites for higher levels, although Thomas believes Levels 2 and 3 may be parallel levels.

Thomas (1980) mostly followed Gagné (1970) for the first five levels, but Gagné, an educational psychologist, focused his highest three levels on tasks (mostly verbal tasks) that were primarily used to study human learning and that were *mostly not usable* with animals. Thomas realized that he could base his highest three levels on Bourne's (1970) approach to concepts and that all of the tasks at Gagné's highest levels could be reduced to Bourne's three levels. Thus, Thomas identified Level 6: Class Concepts as based on Affirmation and Negation. Agreeing with Bourne that concepts based on Conjunctive, Disjunctive, and Conditional rules involved parallel operations, Thomas used them for Level 7: Relational Concepts I. Again, agreeing with Bourne, that Conditionals are prerequisites for Biconditionals, Thomas identified Level 8 as Relational Concepts II based on Biconditionals. *One or more Level 6: Class concepts must be involved to construct tasks that assess Relational Concept use at Levels 7 and 8.* In most instances of problem solving by an animal, whether in the laboratory or in its natural habitat, an animal might use several types of learning from Thomas's hierarchy serially and/or concurrently as the task solution requires.

Class Concepts

To affirm a discriminandum as being an exemplar of a Class Concept, tasks must use trial-unique discriminanda, or if discriminanda are used more than once, evidence of concept learning must be limited to first-trial performances; otherwise, the animal might perform the tasks successfully by rote learning based on trial and error. Thomas (1980) distinguished between Absolute and Relative Class Concept use based on the operational difference that (a) to affirm a discriminandum as being an exemplar of an absolute class concept, the animal *need not compare discriminanda* but (b) to affirm a discriminandum as being an exemplar of a relative class concept, the animal *must compare discriminanda*.

For example, if the concept of interest is "Tree," and, among two or more discriminanda presented to the animal, one is obviously a tree and, if the animal *knows* or *learns* the concept (see quotation from Hayes & Nissen (1971 in the next paragraph) of “Tree,” *it need not compare other discriminanda* to affirm that each new exemplar of a tree is an example of the absolute class concept “Tree.” The most used example of a relative class concept in the animal learning/cognition literature has been the Oddity task. The most common method to study oddity is to present three discriminanda, two of which, the nonodd discriminanda, are identical, and the other discriminandum, the odd one, differs in color form and size from the nonodd discriminanda. To affirm the odd discriminandum, the animal *must compare* all discriminanda.

Before leaving this section, this is a good place to remind all who study use of class concepts by animals of what Hayes and Nissen's wrote in 1971.

> We cannot imagine any set of operations, applied to any subject, that could detect a concept without at the same time operating to induce its formation (Hayes & Nissen, 1971, p.79)

After quoting Hayes and Nissen, Thomas and Ingram (1979, p. 42) added,

> In other words, the acquisition of new concepts and the detection of existing concepts are hopelessly confounded with the subject's acquisition of the reinforcement contingencies, thus the distinction between newly learned and existing concept use is scientifically meaningless.

Most animal cognition investigators, including this writer, have tended to write about animals “learning” concepts when, as just shown, that cannot be known. However, that is not likely to be an easy habit to break. Hayes and Nissen’s statement and Thomas and Ingram’s addition *do not apply* to learning to use relational concepts. Relational concept learning requires using class concepts in relation to other discriminanda which may or may not be conceptual and then determining via experience (learning) over trials what that relationship is. One cannot imagine an animal approaching such a task with a prior knowledge of the relationship being investigated. In other words, an animal cannot preemptively know that there is a relationship between, for example, "triangularity *and* "sameness" or "*if* triangle, *then* sameness" because the relationship contingency in a given test is determined arbitrarily by the experimenter, and it is the animal's task to determine what it is.

Uses of Some Relational Concepts Are So Far Unknown

The complementary rules identified above and the biconditional apparently have never been investigated using nonhuman animals, so these six will not be considered further here. However, these are equally valid and may become relational concepts to be investigated by future animal researchers. Furthermore, it is doubtful whether any animal investigator using animals has determined whether disjunctive relational concepts that conform to the truth table has been determined conclusively; this will be discussed in the next section. Conjunctive and conditional concepts will be considered extensively in later sections.

Disjunctive Relational Concepts

Wells and Deffenbacher (1967) investigated the use of conjunctive and disjunctive concepts by squirrel monkeys, and there may be others unknown to this writer, but it is doubtful if any experiments have conformed to either of the two truth tables for the disjunctive (see below). Wells and Deffenbacher (1967) will be used to examine the issues. Wells and Deffenbacher used 81 discriminanda that varied in size (large, medium, small), shape (triangle, circle, square), color (green, yellow, blue) and patterns within a figure (cross, vertical stripes, cross-hatching). Apparently, these were two-dimensional figures pasted on "the face of" 2" x 2" wooden blocks. Their chosen attributes for the disjunctive were large-square and yellow-cross. There are two types of Disjunctive; one is known as the Inclusive Disjunction (sometimes symbolized in a truth table as **v**) and the other is known as the Exclusive Disjunctive (sometimes symbolized **≠**). Wells and Deffenbacher did not specify which they studied, but one can infer reasonably that they meant the Exclusive Disjunction, and that will be the one diagrammed below. Wells and Deffenbacher's Exclusive Disjunction might be verbalized as "all patterns that are large-square or yellow-cross are correct exemplars."

Exclusive Disjunctive Truth Table Illustrated with Discriminanda

p	q	p ≠ q[1]
T large square	T yellow cross	T Correct
T large square	F not-yellow cross	T Correct
F not-large square	T yellow cross	T Correct
F not large square	F not-yellow cross	F Incorrect

[1] ≠ is the symbol for the exclusive disjunction.

Row 1 shows that any large-square discriminandum or any yellow-cross discriminandum when chosen by the monkeys is correct, while any other discriminanda are not. Row 2 shows that any large-square is correct, yellow-cross is not to be presented, and any other discriminandum there is incorrect. Row 3 shows that any yellow-cross is correct, red-square is not to be presented, and any other discriminandum is incorrect. It is not clear whether Wells and Deffenbacher's procedures included trials that corresponded to row 4, namely to present discriminanda to the monkeys that are not a red-square or a yellow-cross to assess whether the monkeys made no choices on such trials. Assuming that was the case, their research regarding the Disjunctive is regarded here as inconclusive. Although it is unknown whether the truth table for the conjunctive was considered by Wells and Deffenbacher, their demonstration of monkeys' use of a conjunctive rule might be valid. However, the way they reported the results leaves this writer unable to determine whether their squirrel monkeys might have learned the reinforcement contingencies for either or both the conjunctive or the disjunctive by rote.

Misuse of the "Conditional Discrimination" Task

There is a long history of animals being tested on "conditional discrimination" problems and being misinterpreted as providing evidence of the use of a conditional rule. Paraphrasing French (1965), the basic conditional discrimination task involves a set of simultaneously presented discriminanda and a set of successively presented discriminanda; typically, two or more simultaneous discriminanda are used on each trial but only one successive discriminandum is used on each trial. Dozens or more experiments referred to as learning a conditional discrimination as French defined it have been used with animals and humans, but most successful performances can be attributed to rote learning based on trial and error.

An easy example might be to use a red block and a blue pyramid as the simultaneous discriminanda and white or black trays on which the simultaneous discriminanda are presented as the successive discriminanda. The experimenter determines the reinforcement contingencies and, for example, when the objects appear on the white tray, responses to the block might be correct, and when they are presented on the black tray, responses to the pyramid might be correct. There have been too many investigators to reference here (including this author earlier in his career) who interpreted the results of conditional discrimination testing as providing evidence for their subjects having used conditional rules such as “if the tray is white, then the red block is correct” or “if the tray is black, then the blue pyramid is correct” when the task was not constructed to meet the conditions of the truth table for use of a conditional rule and when the task might have been learned by rote.

Maier and Maier (1970) described conditional discrimination as a measure of concept learning. That might be correct in some cases (see two examples below), but most conditional discrimination tasks used with animals do not provide evidence of conceptual conditional discrimination; rather, they likely involved learning by rote. Interpreting such results as reflecting an animal’s use of a conditional rule is at best inconclusive and is most likely incorrect.

An early study using rhesus monkeys and a conditional discrimination task as French (1965) defined it was by Spaet and Harlow (1943). They did not use the term “conditional discrimination;” rather, they referred to their investigation as involving “multiple sign problems.” They administered two tasks. The simultaneous discriminanda in both tasks were the same three identical brass doorbell buttons (hereafter “button” for short) and same three identical T-shaped objects. The successive discriminanda were yellow or black trays on which the simultaneous discriminanda were presented. Spaet and Harlow’s (1943) first task involved oddity problems where a button or a T might be odd and two buttons or two Ts might be nonodd simultaneous discriminanda. If the oddity problem was presented on the yellow tray, the odd object, either the button or the T, was correct, and if the tray was black, either of the nonodd discriminanda, either of two buttons or two Ts, was correct. Food reinforcers were used for correct responses.

Spaet and Harlow were not clear whether the odd object might appear in the left (L), center (C), or right (R) positions versus only on the L or R right positions. If the L-C-R positions were used and, given that the same six simultaneous discriminanda were used, only six specific configurations of object presentations were possible. If only the L-R positions were used, only four specific configurations of object presentations were possible. With only 4 or 6 specific configurations, it is possible the monkeys learned the specific configurations by rote. For the second task, when three buttons appeared on the yellow tray, the correct choice was to the left-most button. When three Ts were presented on the yellow tray, the right-most T was correct. These contingencies were reversed on the black tray. With this task, only four specific configurations were possible. That the monkeys required from 4,320 – 6,840 trials to learn both tasks suggest that they learned the different configurations by rote. Nissen (1951) trained a chimpanzee on 16 concurrently-presented conditional discrimination problems, and it took 15,796 trials to learn them. Nissen was unsure how the animal performed the task, but the following appear in his discussion of how the chimpanzee, Frank, might have performed.

> . . . it might be that each of the . . .[discriminanda] . . . was learned more or less independently (p.14) [four paragraphs later]. It still remains possible, however, that Frank did become responsive to

> . . . a principle but that its expression in overt behavior was obscured by prepotent effects on a “lower level” (p. 15).

Thomas and Kerr (1976) tested three squirrel monkeys on a task involving trial-unique oddity problems as the simultaneous discriminanda, and black or white trays on which to present the oddity problems as the successive discriminanda. Responses to the odd object were correct when an oddity problem was presented on a white tray and responses to either of the nonodd objects were correct when presented on a black tray. Given the use of trial-unique discriminanda, specific configuration learning was not possible. Correct responses resulted in a food reinforcer that was accessible from a food well beneath the correctly chosen discriminandum. Object positions and color of tray used on each trial were varied quasi-randomly. All three monkeys met a criterion of 90% correct responses in 20 successive trials. Thomas and Kerr concluded that the monkeys had demonstrated “conceptual conditional discrimination,” which would be accurate if only “conditional discrimination” as defined by French (1965) was intended. However, Thomas and Kerr (1976) then made the mistake, as many others have done, when they interpreted their monkeys as having used a biconditional rule analogous to Bourne’s (1970) humans subjects’ use of a biconditional rule. Unrealized by them, Thomas and Kerr’s task did not conform to the truth table for the biconditional or for the conditional.

The Conjunctive-Conditional Conundrum

Thomas eventually realized that Thomas and Kerr’s (1976) findings might be explained best by their monkeys using a conjunctive rule, as their task was consistent with the truth table for the conjunctive. Nevertheless, even though their task did not conform to the truth table for the conditional, it remained possible that the monkeys were responding according to using a conditional rule; see more on this below.

Thomas's first opportunity to correct the record was in the Discussion section of a study by Burdyn and Thomas (1984) who investigated "conditional discrimination" as French (1965) defined it. Burdyn and Thomas tested squirrel monkeys and conceptual simultaneous and conceptual successive discriminanda. Trial-unique exemplars of objects manifesting "sameness" or "difference" were used as the simultaneous discriminanda. The successive discriminanda were exemplars of "triangularity" and "heptagonality" (solid triangles and heptagons drawn on white cards); they used a sufficient number of these discriminanda to render rote memorization of specific triangles and heptagons unlikely. However, again like Thomas and Kerr (1976), while Burdyn's and Thomas's monkeys might have used a conditional rule, the experimental design conformed only to the truth table for a conjunctive rule.

Other investigators have reasoned erroneously that animals performed conditional discrimination tasks by means of propositional reasoning such as a conditional rule [e.g., chapter by Jerre Levy (pp. 157-173) and Terrence Deacon's first of two chapters (pp. 363-381) in Jerison and Jerison, (1988)]. Levy and Deacon suggested the significant role that conditional discrimination had in the evolution of language. Levy (p. 164) cited animals' abilities for conditional discrimination as possibly representing "a preadaptation of the simian brain for the evolution of human propositional reasoning," and Deacon (p. 408) referred to the role of conditional discrimination learning in the evolutionary selection for "symbolic communication." Levy based her hypothesis on research by Dewson (e.g., 1977) and Deacon based his on research by Petrides (1987). However, the conditional discrimination experiments by Dewson and Petrides were amenable to rote learning. Dewson made no claims regarding propositional reasoning, but Petrides was clear in his belief that his experiments showed use of a conditional rule; specifically, he referred to "if-then" reasoning when his task did not conform to the truth table for a conditional rule.

Bourne's (1970) Figure 1 shows how human or animals (Bourne only studied humans) might partition nine discriminanda into correct and incorrect examples using conjunctive, disjunctive, conditional, or biconditional rules. In Bourne's Figure 1 (please see Bourne, 1970, p. 548, for his Fig. 1), there were nine discriminanda based on three colors and three shapes (square, triangle and circle) where all shapes might be presented in one each of three colors. As his article was in a black and white print journal, "redness" (Bourne's characterization) was represented when the square, circle, and triangle were drawn with stripes. The other two colors not named in Figure 1 were represented as black-filled or white-filled squares, circles and triangles. To illustrate correct versus incorrect partitioning according to conjunctive, disjunctive, conditional and biconditional truth tables, Bourne used red and square as his focal attributes. These attributes will be the illustrative discriminanda in the truth tables for the conjunctive and the conditional below. The

“&” symbol or the word “and**”** symbolize the conjunctive rule, so p & q is equivalent to saying “p and q.”

Conjunctive Truth Table Illustrated with Discriminanda

p	q	p & q
T red	T square	T Correct
T red	F not-square	F Incorrect
F not-red	T square	F Incorrect
F not-red	F not-square	F incorrect

In partitioning, Row 1 indicates a discriminandum must be red and square to be a correct exemplar. All other discriminanda are exemplars of incorrect discriminanda.

With the conditional, the “>” symbol or the words “ “if-then” symbolize the conjunctive rule, so p > q is equivalent to saying “if p, then q.”

Conditional Truth Table Illustrated with Discriminanda

p	q	p > q
T red	T square	T Correct
T red	F not-square	F Incorrect
F not-red	T square	T Correct
F not-red	F not-square	T Correct

In partitioning correct and incorrect discriminanda according to the conditional rule, row 1 shows discriminandum p must be red and discriminandum q must be square to be correct. Row 2 shows that if discriminandum p is red and discriminandum q is not-square, by implication the red triangle and the red circle are incorrect exemplars. Rows 3 and 4 where p is not-red shows there is no constraint against any not-red discriminanda being correct. This writer is unaware of any animal research purporting to study use of the conditional rule that has incorporated into its experimental design ways to test the contingencies in rows 3 and 4. This writer is not confident that a conditional-rule-use-experiment can be designed that could be used successfully with animals, but that is an open question.

Bourne's (1970) Figure 1 was for demonstration. His actual testing discriminanda included 3 colors (red, green, blue) 3 shapes (square, triangle, circle), 3 sizes (small, medium, large) and 3 variations in number; that is, 1, 2, or 3 discriminanda of a given color, shape and size might be used. For each rule being tested (viz., conjunctive, disjunctive, conditional, or biconditional) the subject was handed a card specifying the pair of relevant attributes being used to partition whether each discriminandum was correct or incorrect. The subject had a two-button device, one for "Correct" and one for "Incorrect" discriminanda according to the rule being used. A light above each button would illuminate if the answer was correct. The subject had to infer the rule over several to many trials based on feedback from the lights whether her/his choice on each trial was correct or incorrect. The criterion for mastery of a given rule was 16 successive correct responses. Based on the number of trials needed to meet criterion, Bourne's subjects revealed the following order of difficulty where **<** symbolizes fewer trials

Conjunctive < Disjunctive < Conditional < Biconditional

An important part of Bourne's evidence was the subject's explanation of the rule that was correctly inferred. Such evidence is not available to animal research Investigators.

Natural or Mental Logic Versus Symbolic or Standard Logic

It would be remiss to fail to consider the longstanding interest by many cognitive scientists in "natural logic" or "mental logic" as opposed to standard truth-table-based logic such as that used in Bourne’s research. Martin D. S. Braine (1926-1996) was among the best-known advocates for “natural” or “mental” logic. Earlier he used the phrase “natural logic” (Brain, 1978) and later he preferred the phrase “mental logic” (Braine & O’Brien, 1998; this was an edited volume published after Braine’s death in which Braine, O’Brien, and others contributed chapters). This literature is too vast and too complicated to summarize here, but relevant points pertaining to animal research are that (a) Braine and others used only human subjects and verbal tasks, usually in the form of providing the premise of a syllogism where the subjects provided the predicate that they conceived to be appropriate and (b) critical to the evidence were the subjects’ verbal explanations. Neither of those are possible with animals. Perhaps, the most important result of this research for animal researchers is that Braine and others concluded that humans can reason correctly, for example, according to a conditional rule without necessarily using tasks that conformed fully to truth table requirements for a conditional rule.

An important implication for animal research is that it is *possible* that when “conditional discrimination” tasks as defined by French (1965) that involve class concept discriminanda are performed successfully by animals, the animals *might* have used a conditional rule. However, it seems unlikely that tasks to confirm such with animals can ever be constructed. Therefore, unless someone can design an experiment using class concept discriminanda and procedures that conform to either disjunctive or conditional truth-table-based rules, the only conclusive evidence of relational concept learning by animals appears to be limited to using conjunctive rules.

Finally, one can do nothing more than speculate about what neural processes an animal might employ to use conjunctive rules successfully. Referring back to Burdyn's and Thomas's (1984) evidence of squirrel monkeys using conceptual conjunctive rules, a human might verbalize externally or internally that "triangle *and* sameness" go together and that "heptagon *and* difference" go together. One can easily imagine a human verbalizing externally or internally the solution to the same task as "*if* triangle, *then* same" and "*if* heptagon, *then* difference." A human can explain her/his conceptualization of the task, but unfortunately a monkey cannot. It is baffling to think how a monkey is able to perform a conjunctive task without having language to express it, even if only internally. Dr. Doolittle, we need you!

Cross-References

Associative concepts

Categorization

Concept formation

Deductive reasoning

Harry Harlow

Learning

Intelligence

Oddity

Problem solving

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